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A PROBABILISTIC MODEL FOR DETERMINING

RESIDENTIAL POSTAL ROUTE SIZE

A THESIS

Presented to

The Faculty of the Graduate Division

by

Steve Voytek Jr.

In Partial Fulfillment

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Master of Science in Industrial Engineering

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A PROBABILISTIC MODEL FOR DETERMINING

RESIDENTIAL POSTAL ROUTE SIZE

Approved:

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SUMMARY

The purpose of this study is to investigate a method of determining residential (single family residences) postal route size. The objective is to develop a mathematical model of a residential route to be used in determining the optimum route size with the criterion of minimizing the total expected cost of idle time and overtime.

An established residential foot route was chosen for the study to obtain the necessary data to formulate the model. The volume of mail for the route was sampled for 30 days to estimate the probability distributions for letters and flats. Flats are pieces of first, second, or third class mail which are larger than letter size mail. Also, the undeliverable mail and the percentage of stops receiving mail were recorded during this time.

Using the estimated mail volume distributions and other work function times estimated by a linear regression method, the model developed was

$$TT = K_1 + K_2\bar{x} + K_3\bar{y} + K_4 - K_5e^{-(\bar{x} + \bar{y})/k}$$

where $\bar{x}$ and $\bar{y}$ are normal independent random variables representing the means per day of letters per stop and flats per stop respectively.

To estimate the distribution function for the above

model, a computer program was written using a Monte Carlo technique. Assuming that the work is performed at a constant rate and the criterion is to minimize the total expected cost of idle time and overtime, the optimal route size, $\hat{N}$, was determined by

$$F(A/\hat{N}) = \int_0^{A/\hat{N}} f(t)dt$$

$$F(A/\hat{N}) = C_o/(C_o + C_I)$$

where f(t) is the density function of the time per stop, and C_I and C_o are the cost of idle time and overtime respectively. Therefore, the optimal $\hat{N}$ is equal to $A/\hat{t}$, where $\hat{t}$ is determined from the distribution function, F(t), and A is the break even point between idle time and overtime.

CHAPTER I

INTRODUCTION

The growth of the Postal Service reflects the growth and development of the United States. During the first fiscal year of operation in 1790 a total of 75 post offices brought in a revenue of $37,935 and experienced an expenditure of $32,140. As of June 30, 1966, there were 33,121 post offices in the United States. The revenue for the 1966 fiscal year amounted to $4,784 million, while an expenditure of $5,726 million was experienced (1).

The United States Post Office Department is concerned with giving the public good service for a reasonable cost. The service has improved but the cost has risen since the time of rural free delivery when an old farmer placed a note beside a crack in his fence which read: "Put the Male Hear." (2). Since a high level of service and low cost are somewhat incompatible, one must look for a compromise between the two.

Approximately one third of the United States Postal employees (190,000) are carriers serving over 54 million families and over 4 million businesses. Since the volume of mail for each route is not deterministic, an interesting problem involving route size determination arises. The time required may be divided into office time and street time. The major portion of office time involves the casing of letter mail and flats (flats are pieces of first, second, or third class mail which are larger than letter size mail), and the forwarding of undeliverable mail. The office time portion will vary from day to day due to the performance of the individual carrier and the volume of mail received. The major portion of street time is the delivery time and it will fluctuate due to the performance of the individual carrier and the variable percentage of residences receiving mail each day.

The casing and delivering of mail could be classified as a repetitive type of work. Thus, an incentive system might be a way to be considered in obtaining higher performance. At the present time, there is a minimum standard office time according to the volume of mail, but there is no incentive for the carrier who demonstrates extremely high performance during the route check week[1]. This can and does lead to more work (larger routes) for the efficient and energetic carrier and less work (smaller routes) for the less efficient and less conscientious carrier as long as the minimum standard office time is met[2].

Presently the United States Post Office Department is considering the feasibility of establishing street standards for

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[1]Inspection of a route is the observation by a supervisor of a carrier's office and street work for an entire day, and includes the counting and recording of the mail handled and the time used for the six consecutive workday periods.

[2]See page 8 for standard office time.

city letter carriers. If office and street time standards can be established, one may consider using these time standards for determining the route size without regard to performance unless an incentive system is used.

Purpose and Objective

The purpose of this thesis is to investigate a method of determining residential (single family residences) postal route size. The objective is to develop a mathematical model of a residential route to be used in determining the optimum size when the criterion is minimizing the expected cost of idle time and overtime.

The thesis will be presented in two parts. The first part will be concerned with the collection and analysis of the data. This will involve the determination of the mail volume distributions, and the relationships between the volume of mail and various work functions. The necessary data to formulate the distributions and relationships were collected by sampling the mail volume of a typical residential route (selected by the supervisor) for thirty consecutive working days.

The second part presents the formulation of a mathematical model. An illustrative example will be stated and solved for the optimum route size. The thesis does not intend to develop or verify the standard times necessary to perform the related work duties. Thus, the standard times used in the model will be standard times established by the United States Post Office Department or reasonable estimates for standard times not available.

The Mail Carrier's Job Description

A mail carrier starts work at about 6:00 A. M. He will spend about two to three hours at the post office arranging the mail for his route in the order in which it will be delivered. The mail has been placed at his sorting rack by clerks.

To do this sorting, the carrier uses a case which is actually an upright box with compartments or pigeonholes labeled with the names of streets, house numbers, or buildings. The carrier, as he sorts, re-addresses mail to be forwarded and marks the mail for those who have moved without leaving a new address. Such mail is returned to the sender.

Once the mail is completely sorted and arranged, the carrier assembles it into bundles which are numbered in the order of delivery. These bundles are then put in their proper order in his satchel to be delivered by foot, truck or mailster[3]. The carrier is allowed to carry only 35 pounds of mail in his satchel for a walking route, and if the accumulated mail amounts to more than this specified weight, the excess mail is placed in storage boxes along the route by mail truck drivers.

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[3]A mailster is a ¼ ton, three-wheel vehicle used for mail delivery.

As the mail carrier goes from door to door, he places the flats and letter mail in the individual mail boxes at the front entrance. The carrier collects the money on postage-due mail or on C.O.D. parcels, and he gets signed receipts for registered or insured mail (3).

CHAPTER II

LITERATURE SEARCH

Present Procedure For Determining The Route Size

Route inspection and a count of mail volume is conducted on each route, each fiscal year, during normal mail volume periods between the first week of September and May 31, excluding December.

The count of mail is the physical counting and recording of the number of pieces of mail delivered on a city delivery route each day during the six day route inspection period.

The data obtained during the week of counting are used to estimate the following:

a. The volume of mail handled.

b. The amount of office and street time used by the carrier.

c. The efficiency with which the carrier performs his work.

d. The adequacy of service to the public.

The above data are used to attempt to balance the workload for each route as near as possible to an eight hour workday for the individual carrier.

Office time is the total time spent in the office before and after serving the route. Under normal conditions, the carrier's office time is fixed at the average time required to perform office work during the count week but not in excess of the average minimum standard office time as established by the Post Office Department[4].

Street time is the total time spent outside the office during the tour, exclusive of lunch period. The carrier's street time is recorded each day (on a time card) by the carrier and it is only verified by actual inspection one day. Ordinarily the carrier's street time will be the net time used on the heaviest day (the day on which the largest total number of pieces of mail of all classes was delivered and not the day on which the carrier used the most street time) during the week of the count. Unless the heaviest mail volume day was the day of inspection, the above street time is the time recorded by the individual carrier. If this street time results in considerable undertime on lighter days, the street time is changed to the net time used on the next-to-the-heaviest day of the route check week.

The office time and street time as described above are added together to give the total time for the route. If this is close to eight hours, the route is not adjusted; however, if the route is in need of adjustment, the following method is usually used. The net office time is divided by the total deliveries to

[4]An exception will be made for carriers who have served continuously for 25 years or more or are over 55 years of age provided the carrier's conduct is satisfactory.

obtain the office time per delivery.[5] The to and from travel time is deducted from the street time and the remaining time is divided by the deliveries to obtain the street time used per delivery. The above average office and street time per delivery are added and then multiplied by the number of deliveries to be added or subtracted to determine the total allowance time involved. This allowance time is then added to or subtracted from the route involved to make as nearly as possible an eight hour route.

Minimum Time Standards For Carrier Office Work[6]

1. Casing Mail	
(a) letters, cards, and letter-size circulars	1 min./18 pcs.
(b) all other mail (Flats)	1 min./8 pcs.
2. Registered, certified, postage due, C.O.D., customs-due, signing for, returning funds and receipts, and for completion of Form 3849	Actual time
3. Getting and returning box keys	Actual time

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[5]The definition of a "delivery" and a "stop" is the same for a residence (a single family dwelling). However, an apartment building where mail is delivered to receptacles grouped together is one possible stop and as many possible deliveries as there are receptacles. A duplex with entrances and boxes separated, counts as two possible stops and two possible deliveries.

[6]Supervision of City Delivery Service, Methods Handbook, Series M-39, U. S. Post Office Department, Washington, D. C., 1966, Exhibit 216.531.

4.	Withdrawing Mail	Actual time
5.	Answering official communications	Actual time
6.	Loading (mounted routes only)	Actual time
7.	Strapping out time (when mail must be placed in order of delivery)	Actual time
8.	Insured mail entered in Form 3883	2 min./5 pcs.
9.	Second class marked up	1 min./2 pcs.
10.	Form 3579 (undeliverable 2nd class matter)	2 min./pc.
11.	Pieces marked up (1st, 3rd, and 4th class)	1 min./4 pcs.
12.	Pieces of all classes of mail separated for forwarding or returning	1 min./10 pcs.
13.	Insured receipts turned in	1 min./pc.
14.	Change of Address recorded in route book	2 min./pc.
15.	Facing mail collectings in office	1 min./40 pcs.
16.	Personal needs	5 minutes
17.	Strapping mail in bundles, preparing relays and placing mail in satchel	1 min./70 pcs.

The Effects of the Present Method of Determining the Route Size

The carrier's performance during route check week is an important factor in establishing the route size. Therefore, each time a carrier bids on a route, the route will probably be readjusted to the new carrier's performance. Routes are also adjusted due to building construction or mail volume changes[7].

An established residence area is considered to be static, in regards to significant building constructions and mail volume changes. Thus, a route change in this area is probably due to the performance change of the individual carrier. A route adjustment affects the following:

(a) The route itself.

(b) Neighboring routes.

(c) The clerks who must relearn the route scheme[8] for distributing the mail.

(d) The patron's time of receiving the mail.

Therefore, if routes were set at standards regardless of the carrier's performance, the number of route adjustments would be substantially reduced.

At the present time, the route size is determined so that each carrier has an eight hour day using the individual's average office time and the street time recorded by the individual for the heaviest mail volume day during the week of inspection. In general, this does not minimize the expected cost of idle time and overtime for the year, excluding the Christmas season.

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[7]A carrier throughout the year can bid for any open route, resulting from a retirement, new established route, or a transfer of a carrier, on the basis of seniority. Once the route is secured, it is the carrier's as long as he desires to keep it.

[8]A route scheme is a systematic plan to guide the effective distribution of mail to the designated routes for a particular post office.

The mail volume throughout the year is affected by seasonal changes. The most marked changes occur in the summer months and at Christmas time. During the summer months the mail volume may decrease a small amount for certain areas, while at Christmas time there is a substantial increase. The increased mail volume during the Christmas season is handled by hiring seasonal employees, and giving overtime to regular and substitute carriers. During this time the regular carrier may case mail five to six hours and deliver part of the route, while the substitutes and temporary carriers deliver the remaining part of the route. The regular carrier may work 10 to 12 hours per day during the two weeks preceeding Christmas Day.

Excluding the Christmas season, the overtime throughout the year is not given to regular carriers. Substitute carriers[9] are employed for this work. One may question the practice of giving the substitute carriers the work at regular wage rate compared to giving the regular carrier the work at an overtime wage rate. It has been argued that the cost for the above work would be equal or less if done by the regular carrier instead of the substitute carrier on the basis that the substitute must use traveling time to and from the route, office time preparation, and possibly a less efficient mode of transportation.

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[9]A substitute carrier does not have a regular route, nor is the carrier guaranteed an eight hour workday. If the substitute carrier is scheduled to work, then he is assured of at least two hours of work.

The above additional time, along with the fact that the substitute is generally not as familiar with the territory (which usually means that a longer delivery time will be required), will usually result in a higher cost than had the regular carrier performed the work on overtime. This problem has been limited, to some extent, by permitting the supervisor to use judicial overtime.

There is some doubt as to whether a one week sample provides an adequate estimate of the volume of mail for the entire year. Due to the unpredictable volume of advertisements, an extreme one week count could affect the office time considerably, whereas, a partial sampling over a month's time might give a more realistic picture of the mail volume for the particular route.

By using the average office time for the year, variation in idle time and overtime is not considered. The street time and office time for each route are not independent. The important factor influencing street time is the percentage of stops made. Although there is some relationship between volume of mail and percentage of stops, it is not necessarily true that the largest volume of mail always results in the highest per cent of stops, nor is the relationship a linear one.

It would be expected that the street time for the heaviest mail volume day would be greater than the average street time during the week of inspection. However, this is difficult to verify, since the actual delivery time is observed only once during the week of inspection. If the above is true, then what is called an eight hour workday is, in effect, less than eight hours work on the average.

This method of determining street time provides some sort of cushion to the carrier when the mail is heavy and reduces some of the overtime. Thus, the present procedure used by the United States Post Office Department does provide some idle time to compensate for the heavy mail volume days.

CHAPTER III

THE DATA

Given a residential area from which a route of size N residences is to be formed, the following information would be essential:

(a) The probability distribution of the volume of letters.

(b) The probability distribution of the volume of flats.

(c) A relationship between the time to forward undeliverable mail and the volume of mail.

(d) A relationship between street time and office time.

Data Notation

N = Total possible stops for a given route.

n = The number of possible stops sampled.

x_{ij} = The number of letters for stop j on day i.

$\bar{x}_i$ = $1/n \sum_j^n x_{ij}$, the mean number of letters/stop on day i.

$\bar{X}_i$ $\cong$ $N \cdot \bar{x}_i$, the estimated total volume of letters for N stops on day i.

y_{ij} = The number of flats for stop j on day i.

$\bar{y}_i$ = $1/n \sum_j^n y_{ij}$, the mean number of flats/stop on day i.

$\bar{Y}_i$ $\cong$ $N \cdot \bar{y}_i$, the estimated total volume of flats for N stops on day i.

z_{ij} = $x_{ij} + y_{ij}$, the number of pieces for stop j on day i.

$\bar{z}_i$ = $\bar{x}_i + \bar{y}_i$, the mean number of pieces/stop on day i.

$\tilde{Z}_i \cong N \cdot \bar{z}_i$, the estimated total number of pieces/stop on day i.

p_i = The percentage of stops made out of n possible stops on day i.

$P_i \cong N \cdot p_i$, the estimated total percentage of stops made out of N possible stops on day i.

Data Collection

The data were collected on an established residential[10] foot route at station "A" in Atlanta, Georgia. The route consisted of 322 possible residential stops and 14 possible business stops. The ratio of deliveries per stop for the residential area was approximately one (336/322 = 1.04); therefore, a "stop" and a "delivery" will be considered to be equivalent for this discussion. Ten stops were omitted from the 336 possible stops since seven were vacant at the time and three businesses thought to have a large volume of mail (compared to a residential stop) were sampled separately.[11] The data were obtained from fifty stops randomly selected out of the remaining 326 stops, providing a sampled fraction of 15.3 per cent of the above residential area.

Assuming that $\bar{x}_i$ is drawn from a normal distribution, the 95 per cent confidence interval for the mean number of

[10]The Post Office Department defines a residential route as a foot or mounted route on which 70% or more of the possible deliveries are residential. A foot route does not deliver the route's parcel post whereas a mounted route does.

[11]Vacant residences at the time of route inspection are considered to be possible stops. The ten stops were omitted here because the purpose of this data collection is to estimate the type of mail volume distributions for a residential area.

letters per stop per day is:

$$\bar{x}_i = \bar{\bar{x}}_i \pm t_{\alpha/2,n-1} s_{\bar{x}}/\sqrt{n}$$
$$\bar{x}_i = 1.41 \pm 0.18 \text{ letters/stop}$$

where $\bar{\bar{x}}_i$ is the unbiased estimate of the population mean and $s_{\bar{x}}$ is the unbiased estimate of the population standard deviation calculated by

$$s_{\bar{x}} = \left(\frac{\sum_1^n \bar{x}_i^{\,2} - (\sum_1^n \bar{x}_i)^2/n}{n - 1} \right)^{\frac{1}{2}} = 0.488$$

with n = 30 days. The above procedure is repeated to calculate the 95 per cent confidence interval of $\bar{y}_i$, with $\bar{\bar{y}}_i$ equal to 0.787 and $s_{\bar{y}}$ equal to 0.453 as follows:

$$\bar{y}_i = 0.79 \pm 0.17 \text{ flats/stop} .$$

The 95 per cent confidence level for p_i can be calculated by the formula:

$$p_i = \bar{\bar{p}}_i \pm Z_{\alpha/2} \cdot (p_i(1 - p_i)/n)^{\frac{1}{2}}$$

where p_i is an estimate of $\bar{\bar{p}}_i$. Using an estimated mean $\bar{\bar{p}}_i$ of 0.826, the confidence interval is

$$p_i = 0.83 \pm 0.14 \quad .$$

In addition to recording the number of letters per stop and flats per stop for the above 50 stops, the undeliverable mail for the entire route (336 stops) was recorded. The undeliverable mail, according to class type, is summarized in Table 7 of the Appendix, along with the percentage of stops receiving mail and the mean number of pieces per stop for the 30 working days.

Data Summarization

The data for $\bar{x}_i$ and $\bar{y}_i$ are summarized in Table 1 and Table 2 respectively for 30 consecutive working days.

Table 1. The Mean Number of Letters Per Stop By Days

Days/Weeks	I	II	III	IV	V	$W_{i.}$
T	1.00	1.54	1.20	1.04	1.32	6.10
W	0.90	0.88	1.72	1.72	1.18	6.40
Th	1.38	2.98	1.80	1.22	1.12	8.50
F	1.26	2.10	2.16	1.34	1.32	8.18
S	1.56	1.42	1.34	1.68	1.26	7.26
M	0.62	0.98	1.04	2.24	1.00	5.88
$W_{.j}$	6.72	9.90	9.26	9.24	7.20	42.32
$\bar{x}_{.j}$	1.12	1.65	1.54	1.54	1.20	

$\bar{\bar{x}}_{..} = W_{..}/N = 42.32/30 = 1.41$ letters/stop

Table 2. The Mean Number of Flats Per Stop By Days

Days/Weeks	I	II	III	IV	V	$W_{i.}$
T	1.32	1.02	0.56	1.32	1.28	5.50
W	1.46	1.70	0.88	1.74	0.80	6.58
Th	0.26	0.84	1.44	0.42	0.44	3.40
F	0.34	0.60	0.54	0.62	0.58	2.68
S	0.24	1.20	0.32	0.68	0.20	2.64
M	0.70	0.72	0.18	0.52	0.70	2.82
$W_{.j}$	4.32	6.08	3.92	5.30	4.00	23.62
$\bar{y}_{.j}$	0.72	1.01	0.65	0.88	0.66	

$$\bar{\bar{y}}_{..} = 23.62/30 = 0.79 \text{ flats/stop}$$

The frequency histograms of $\bar{x}_i$ and $\bar{y}_i$ are shown in Figure 1 and Figure 2 with intervals of 0.20 letters per stop and 0.20 flats per stop respectively.

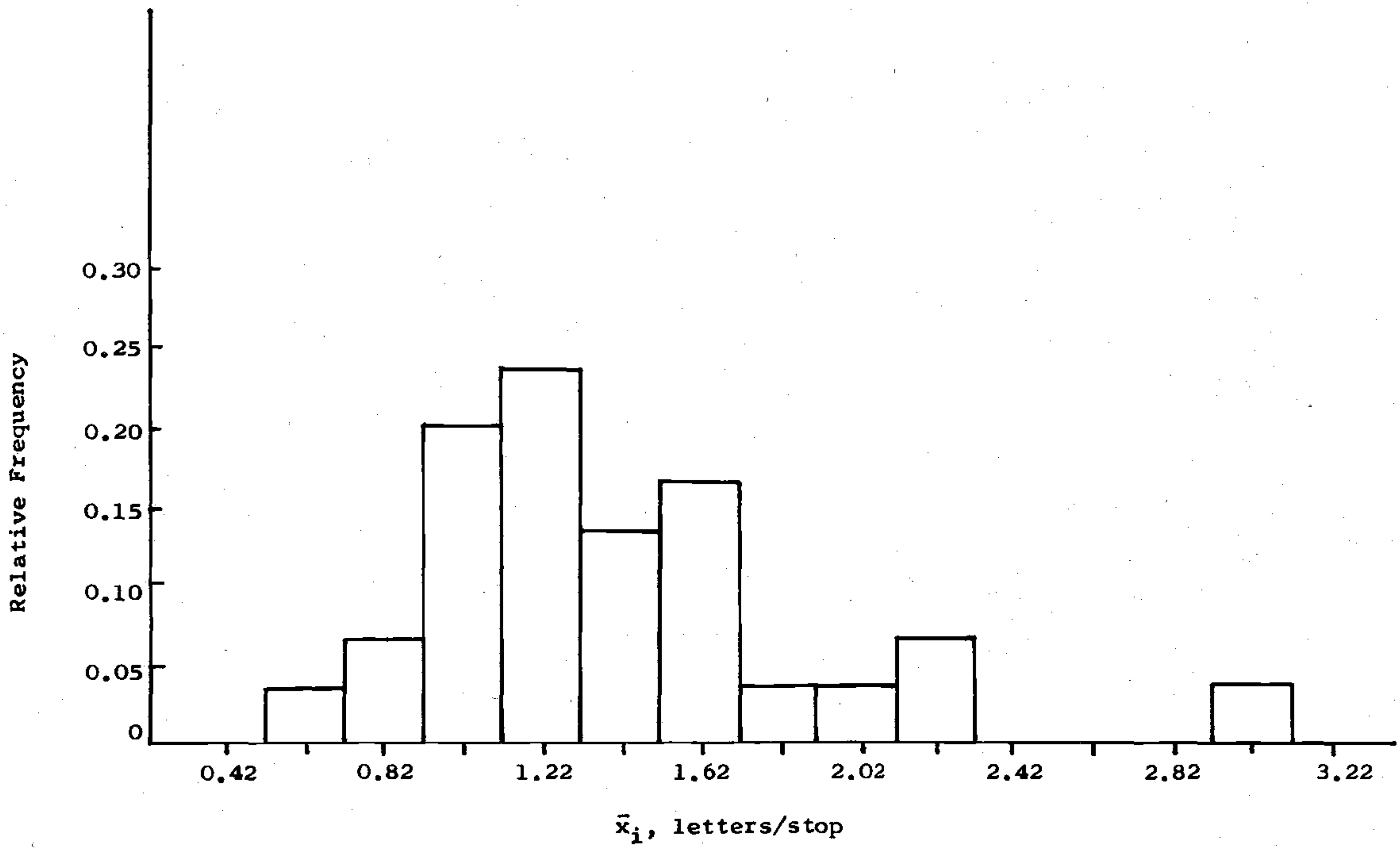

Figure 1. Frequency Histogram of Means Per Day of Letters Per Stop

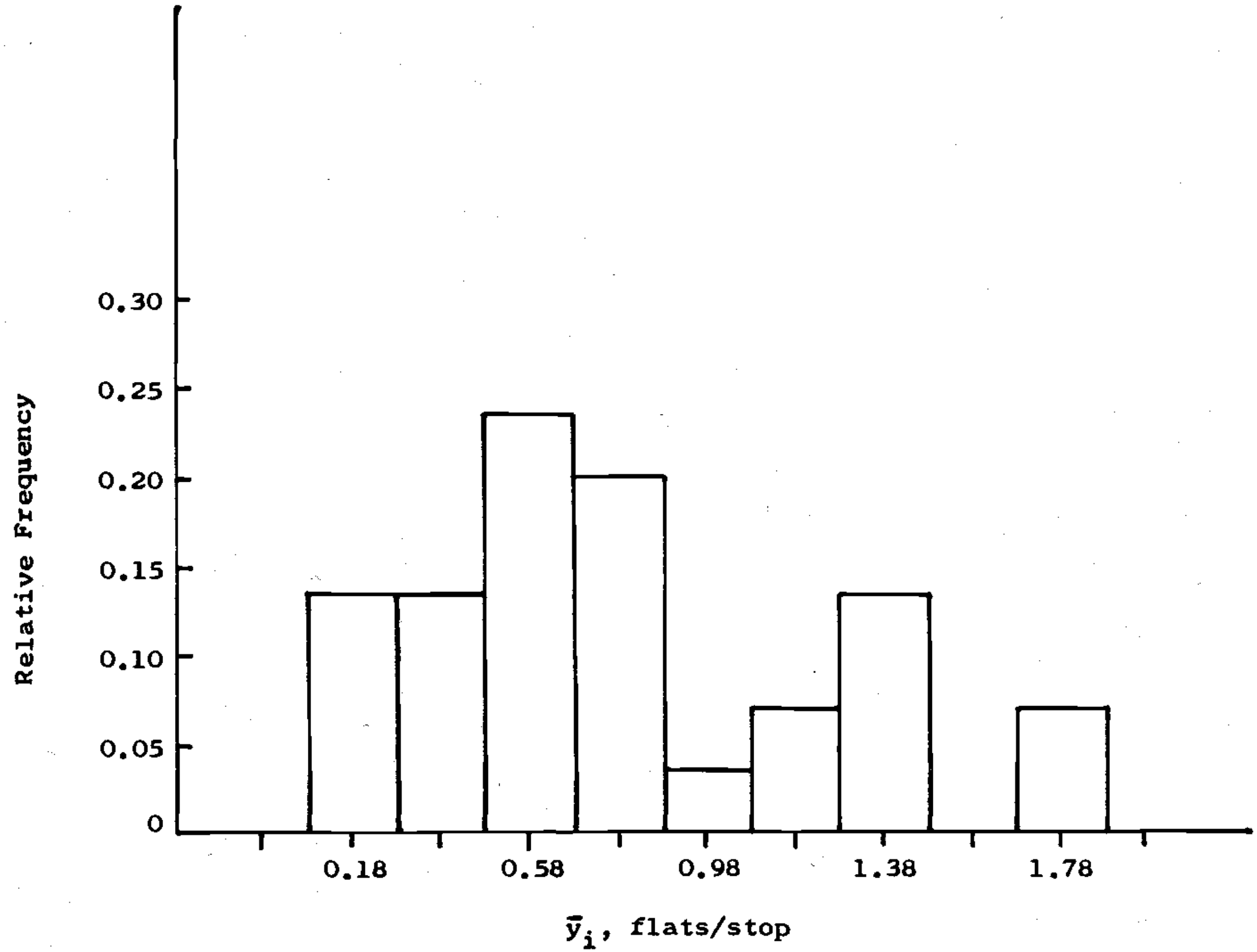

Figure 2. Frequency Histogram of Means Per Day of Flats Per Stop

CHAPTER IV

ANALYSIS OF DATA

Mail Volume Differences Among Weeks

The present procedure for estimating the mail volume for a route is to observe a one week sample. Thus, an interesting question is, how much variation is there between weeks in estimating the true volume of mail? Since the past mail volume data for residential routes are not available, an experiment of randomly selecting weeks and comparing the volume of mail for each week cannot be performed. However, an analysis of variance was performed on the five sample weeks, with the following model equation:

$$\bar{x}_{ij} = \mu + W_j + D_i + \epsilon_{ij}$$

where $\bar{x}_{ij}$ represents the i-th day observation on the j-th week. μ is a common fixed parameter for the whole experiment, W_j represents the effect of the j-th week, D_i represents the day effect and ϵ_{ij} represents the random error present in the i-th observation on the j-th week. The error term, ϵ_{ij}, is assumed to be normally and independently distributed random effect whose mean value is zero and whose variance is the same for all weeks. If an interaction term, DW_{ij}, exist, it cannot be isolated from the error term without replications (4).

The analysis of this model is a two-way analysis of variance, but the main objective is to test the mean week differences. Each week consisted of six consective working days with the experiment being performed for five consective weeks.

The analysis of variance performed on the data of Table 1 revealed the following table for the means of letters per stop for each week. (See Appendix for calculations)

Table 3. Analysis of Variance for Means of Letters Per Stop

Source of Variation	df	Sum of Squares	Mean Square
Between Weeks, W_j	4	1.32	0.330
Between Days, D_i	5	1.22	0.244
Error, ϵ_{ij}	20	4.35	0.218
TOTALS	29	6.89	

To test the hypothesis, H_o: $W_1 = W_2 = W_3 = W_4 = W_5$, where W_j is the mean number of letters/stop for week j, the test statistic is

$$F_{4,20} = 0.330/0.218 = 1.51$$

which is not significant for the F test at 5% or 10% significance level. Thus, the null hypothesis of equal means of letters per stop for each week is not rejected.

Using the same model and procedure on the data of Table 2, the following table was revealed for the means of flats per stop for each week.

Table 4. Analysis of Variance for Means of Flats Per Stop

Source of Variation	df	SS	MS
Between Weeks, W_j	4	0.58	0.145
Between Days, D_i	5	2.84	0.568
Error, ϵ_{ij}	20	2.52	0.126
TOTALS	29	5.94	

To test the hypothesis, $H_o : W_1 = W_2 = W_3 = W_4 = W_5$, where W_j is the mean number of flats/stop for week j, the test statistic is

$$F_{4,20} = 0.145/0.126 = 1.15$$

which is not significant for the F test at 5% or 10% significance level. Thus, the null hypothesis of equal means of flats per stop for each week is not rejected.

The tabulated value for $F_{4,20}$ at the 10% significance level is 2.25. Although the hypotheses of equal means for mail volume among the five weeks was not rejected, the analysis involved the assumption of no interaction between days and weeks. However, the graph of $\bar{x}_i$ and $\bar{y}_i$ versus the weeks in Figure 3 indicates some interaction between days and weeks, since the curves are not parallel. Thus, it is possible that the error

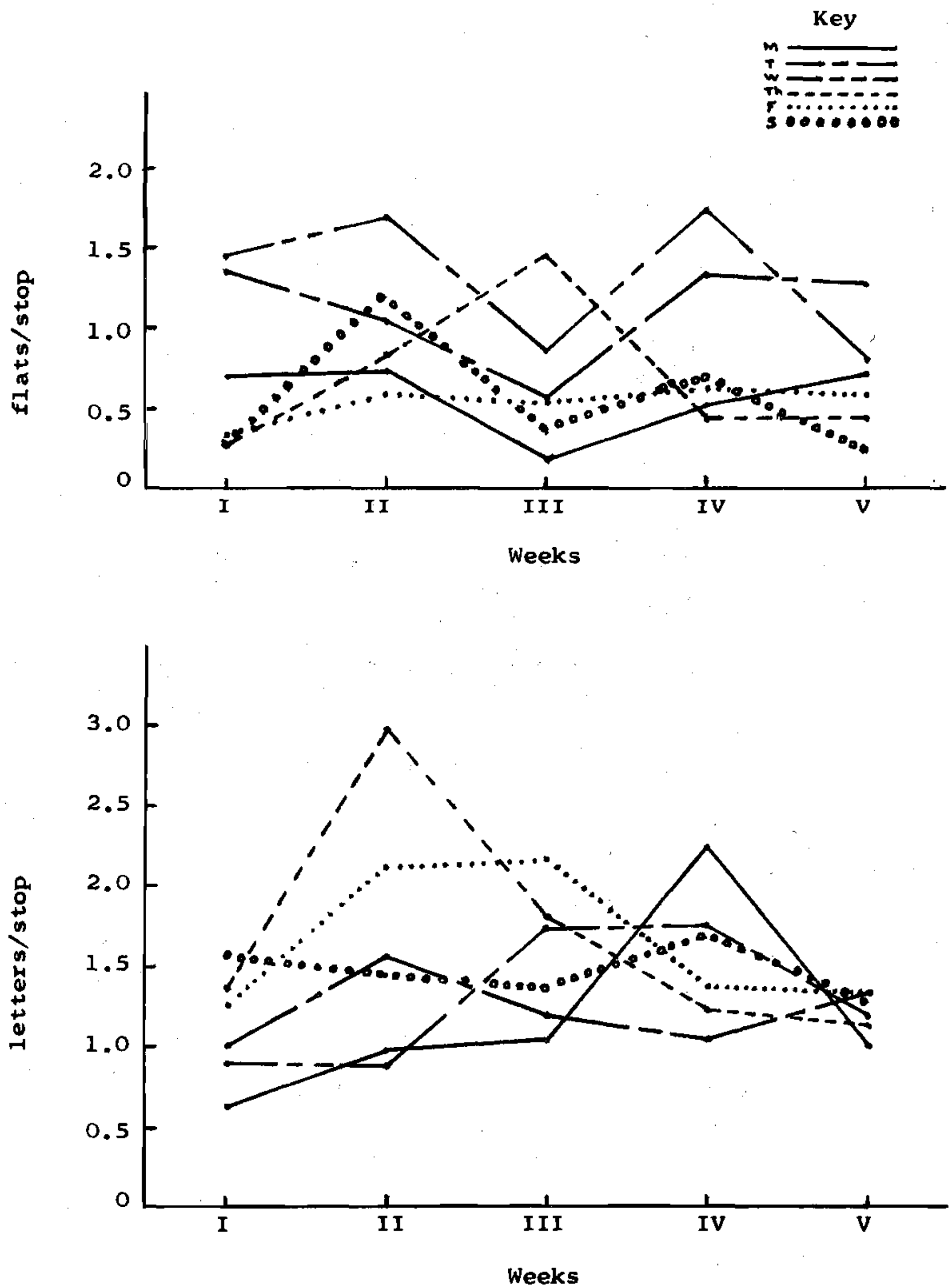

Figure 3. Interaction Between Days and Weeks for Mail Volume

term, ϵ_{ij}, was inflated, due to an interaction term, which resulted in not rejecting the hypotheses.

Mail Volume Distributions

The frequency histogram of $\bar{x}_i$, Figure 1, suggest that the data approximates a normal distribution. Table 5 reveals the observed and expected values of $\bar{x}_i$, using the maximum likelihood estimates of μ and σ in the normal model as $\bar{\bar{x}}_{..} = 1.41$ and $s_{\bar{x}} = 0.488$ letters per stop respectively. The sum of the probabilities in Table 5 differ from one due to rounding errors only.

The χ^2 test for goodness of fit was employed with seven degrees of freedom(5). The calculated value of χ^2 was 8.47 which is not significant for the right tail of the χ^2 distribution with seven degrees of freedom at a 5 per cent significance level. Hence, the hypothesis that the data come from a normal distribution is not rejected.

The same test procedure was calculated for the frequency histogram of $\bar{y}_i$, Figure 2. The estimates for the mean and standard deviation were 0.787 and 0.453 flats per stop respectively, with the results in Table 6. The χ^2 was 6.28 with 5 degrees of freedom which is not significant for the χ^2 distribution at a 5 per cent significance level. Thus, the hypothesis that the data, representing the means per day of flats per stop, come from a normal distribution is not rejected.

Table 5. Goodness of Fit Test for Means of Letters Per Stop

Mean $\bar{x}_i$	0.62	0.80	1.02	1.22	1.42	1.62	1.82	2.02	2.22	2.42	...	3.02
Observed Frequency	1	2	6	7	4	5	1	1	2	0	0	1
End Point	0.725	0.925	1.125	1.325	1.525	1.725	1.925	2.125	2.325	2.525		
Deviation From Mean	-0.686	-0.486	-0.286	-0.086	0.114	0.314	0.514	0.714	0.914	1.114		
Std. Dev. From Mean	-1.41	-0.99	-0.59	-0.02	0.23	0.64	1.05	1.46	1.87	2.28		
Probability	0.080	0.082	0.117	0.151	0.162	0.148	0.114	0.075	0.041	0.031		
Expected Frequency	2.38	2.46	3.50	4.54	4.87	4.44	3.43	2.24	1.24	0.33		
Contrib. To χ^2	0.800	0.086	1.786	1.333	0.155	0.071	1.726	0.686	0.467	1.355		

Table 6. Goodness of Fit Test for Means of Flats Per Stop

Mean $\bar{y}_i$	0.18	0.38	0.58	0.78	0.98	1.18	1.38	1.58	1.78
Observed Frequency	4	4	7	6	1	2	4	0	2
End Point	0.285	0.485	0.685	0.885	1.085	1.285	1.485	1.685	
Deviation From Mean	-0.502	-0.302	-0.102	0.098	0.298	0.498	0.698	0.898	
Std. Dev. From Mean	-1.11	-0.67	-0.23	0.22	0.66	1.10	1.54	1.98	
Prob-ability	0.134	0.118	0.158	0.178	0.158	0.119	0.074	0.062	
Expected Frequency	4.02	3.54	4.75	5.35	4.74	3.57	2.22	1.86	
Contrib. To χ^2	0.000	0.060	1.066	0.079	2.951	0.690	1.427	0.011	

Relationship Between Letters and Flats

It is reasonable to assume that the volume of letters and the volume of flats received for a route each day are independent. This assumption is partially supported by observing the scatter diagram of $\bar{y}_i$ and $\bar{x}_i$ in Figure 4.

Assuming $\bar{x}_i$ and $\bar{y}_i$ were sampled from a bivariate normal distribution, the correlation coefficient calculated by

$$r = \frac{n \sum_i \bar{x}_i \bar{y}_i - \sum_i \bar{x}_i \sum_i \bar{y}_i}{\left(\left[n \sum_i \bar{x}_i^{\,2} - \left(\sum_i \bar{x}_i \right)^2 \right] \left[n \sum_i \bar{y}_i^{\,2} - \left(\sum_i \bar{y}_i \right)^2 \right] \right)^{\frac{1}{2}}}$$

was equal to -0.067. To test the null hypothesis that the population correlation coefficient is zero, the following test statistic was calculated (6)

$$t = r(n-2)^{\frac{1}{2}}/(1 - r^2)^{\frac{1}{2}} = -0.354$$

Referring to the Student's t for n = (30 - 2) = 28 df, one can see that we fail to reject the null hypothesis.

Relationship Between Street Time and Office Time

If at least either one letter or one flat or both is received for a stop, then the above will constitute a stop made by the carrier. An interesting scatter diagram, Figure 5, would be p_i versus $\bar{z}_i$, to estimate the percentage of stops receiving mail. Using the estimate of p_i, the street time can be calculated.

The model for estimating p_i, the percentage of stops to be made, as a function of $\bar{z}_i$, has been taken as an exponential

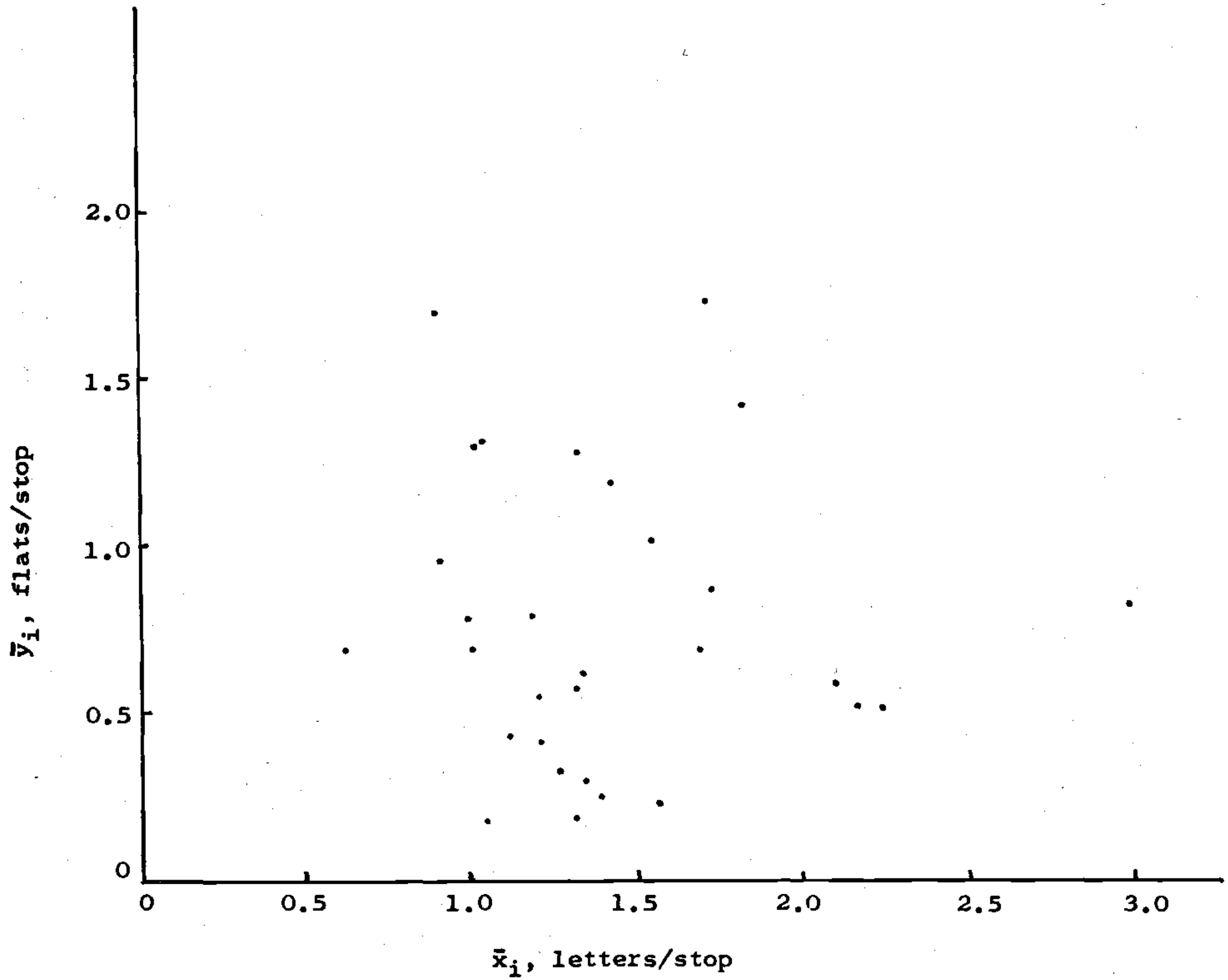

Figure 4. Scatter Diagram of Letters Per Stop and Flats Per Stop

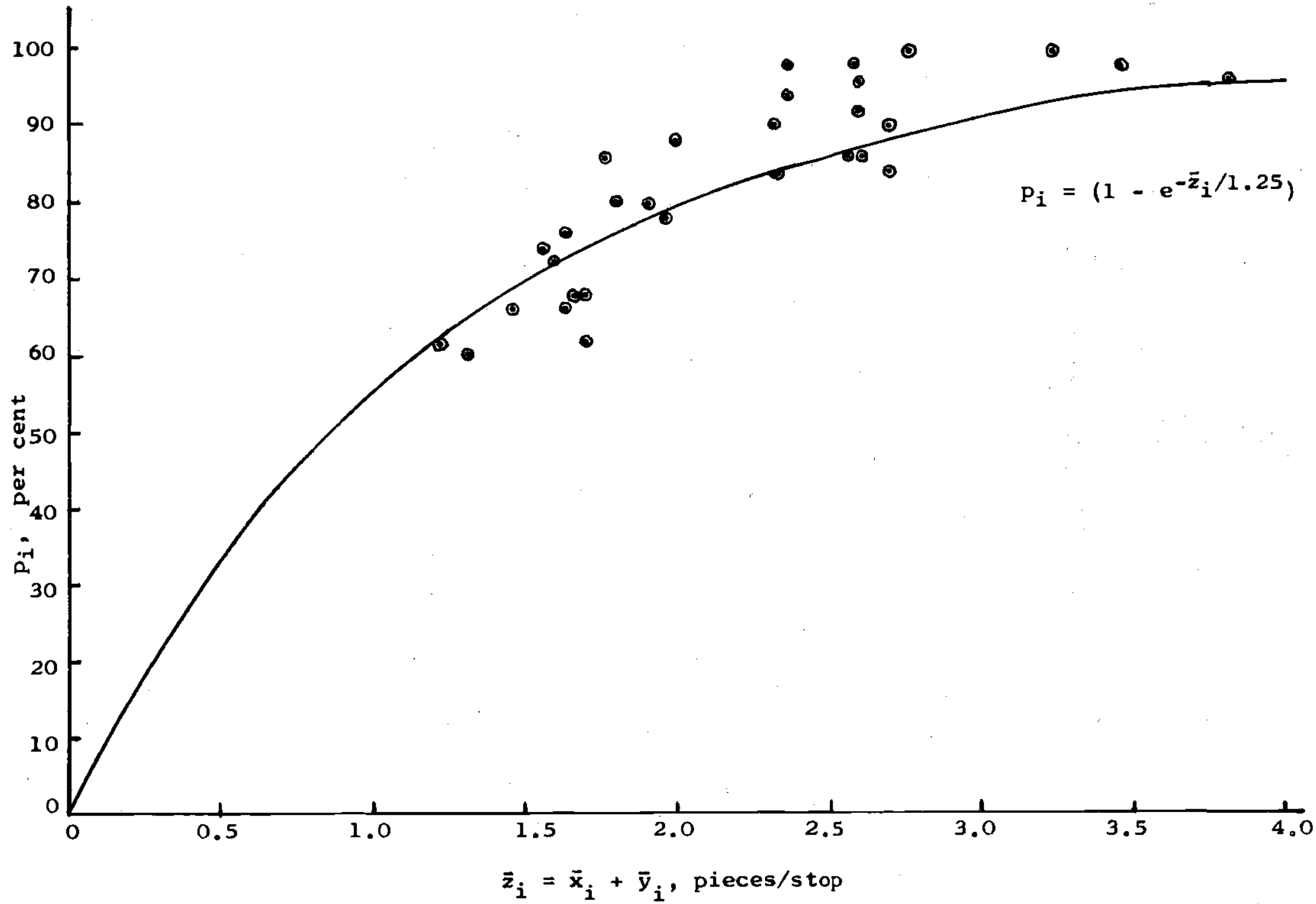

Figure 5. Scatter Diagram of Pieces Per Stop and Percentage of Stops Made

form with the following equation:

$$p_i = (1 - e^{-\bar{z}_i/k}) \quad .$$

By using the estimated grand means for p_i and $\bar{z}_i$, as .826 and 2.196 respectively, k was determined to be 1.25. This model is reasonable on intuitive grounds, since as the volume of mail increases the per cent of stops made can only approach the 100 per cent value. Therefore, by knowing the mean number of pieces per stop, the percentage of stops made can be estimated and in turn, the estimated street time calculated.

Relationship Between Undeliverable Mail and Volume of Mail

An important factor in determining office time is the time to forward the undeliverable pieces of mail. Table 7 in the Appendix shows the undeliverable mail by class type for the entire route along with the standard time, w_i, to forward and separate the undeliverable mail. To investigate the relationship between undeliverable mail and total volume of mail, a scatter diagram, Figure 6, was plotted.

Assuming a linear model of the form $Q_i = m\bar{z}_i$, the regression coefficient, calculated by the least squares method, was

$$m = \sum_i \bar{z}_i Q_i / \sum_i \bar{z}_i^2 = 18.86$$

where Q_i is the undeliverable mail for day i. This line was superimposed on the scatter diagram in Figure 6. It should be pointed out that the independent variable was an estimate of the

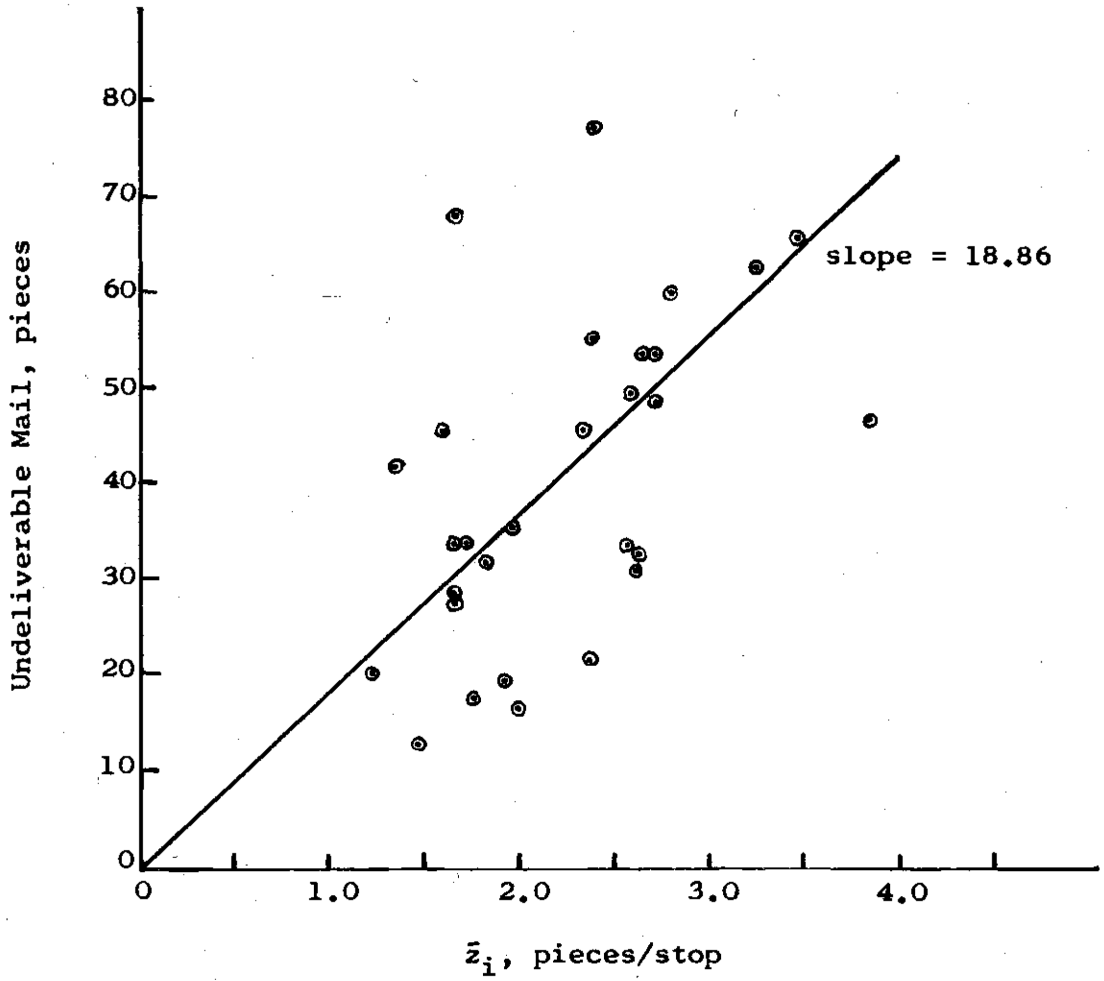

Figure 6. Scatter Diagram of Total Undeliverable Mail and Pieces Per Stop

total volume of mail while the dependent variable was the actual count for the entire route. Although the above would result in some error, the large deviations from the least square regression line are due to the address changes occurring over the period of study, resulting in a fluctuation of the undeliverable pieces of mail. Also, the type of mail will cause some of the large deviations since the companies mailing advertisements and magazines are not aware of address changes, whereas the utility companies are usually informed by the addressee moving in or out of a residence.

The standard time to forward the undeliverable mail depends not only on the amount of mail but also on the class type of mail. Therefore,the standard time required to forward and separate the above mail according to the class types was calculated using the office time standards (page 8, items 9, 11, and 12) and plotted on the scatter diagram, Figure 7. The correlation coefficient between $\bar{z}_i$ and w_i, where w_i is the standard office time to work the undeliverable mail on day i, was $r = 0.498$. Referring to a table of confidence limits for the population correlation coefficient, the 95 per cent condience interval of ρ was approximately 0.16 to 0.72 for a sample of thirty (7). A linear regression model of the form $w_i = c\bar{z}_i$ was used to estimate w_i as a function of $\bar{z}_i$. The regression coefficient calculated by the least squares method was $c = 6.76$. Thus, the regression line superimposed on the scatter diagram in Figure 7 is given by $w_i = 6.76\ \bar{z}_i$ in minutes for $N = 336$ stops, for this particular

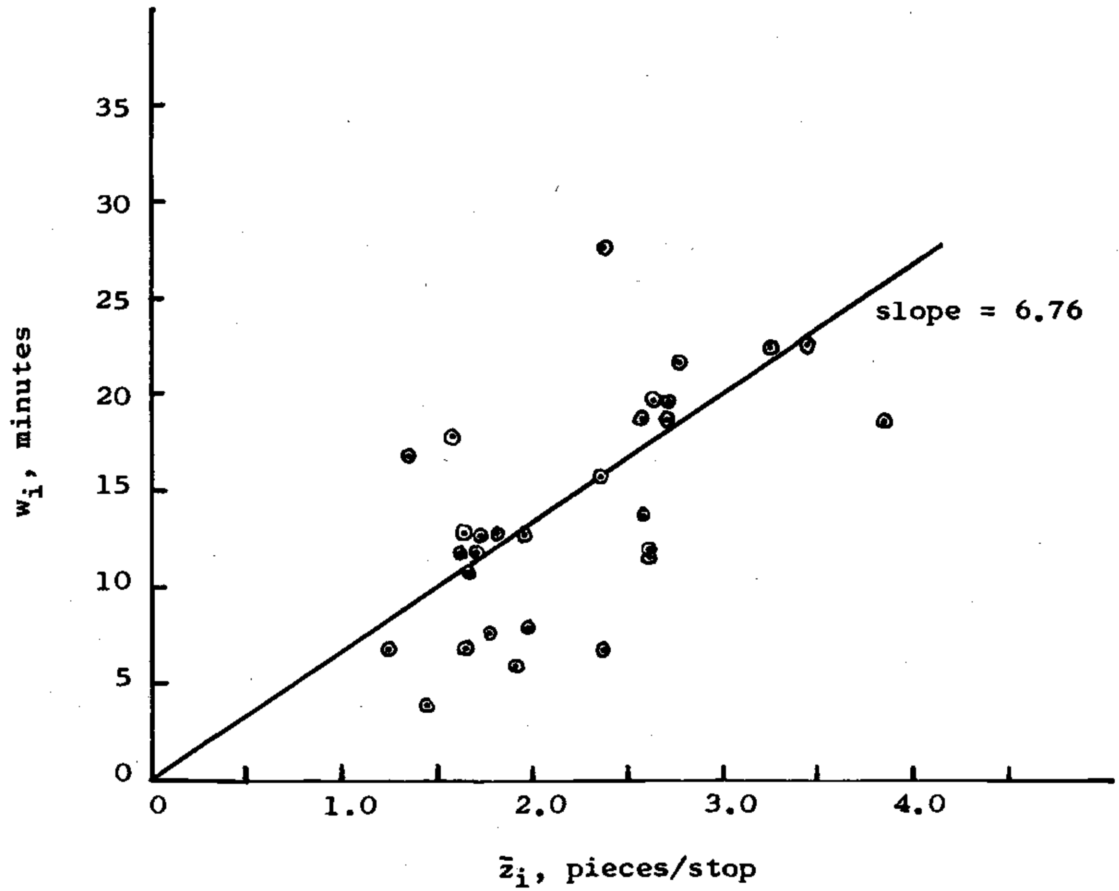

Figure 7. Scatter Diagram of Standard Time To Work Undeliverable Mail and Pieces Per Stop

route. The large deviations from the least square line are due to the reasons discussed in connection with the deviations in Figure 6.

Thus, one is confronted with either selecting an average time per day to work undeliverable mail or use a linear model for estimating the time as a function of $\bar{z}_i$, accounting for the large deviations.

CHAPTER V

THE MODEL DEVELOPMENT

A mathematical model was developed for a residential route within the framework of the following assumptions:

1. The residential area is homogeneous, that is there are no large businesses and the delivery service is either door service or curb service for the area.
2. The ratio of deliveries per stop is approximately one (single family residences).
3. The type of transportation and pattern of delivery is the same each day.
4. Mail arriving each day during office time is delivered that day.
5. The various work functions are performed at a constant rate.

Office Time

The office time can be separated into five major areas as follows:

(a) Time to case letter mail.

(b) Time to case flat mail.

(c) Time to prepare route such as getting and returning keys, checking out special mail, withdrawing mail, loading mounted routes, answering official calls and personal needs.

(d) Time for upkeep of the route such as forwarding undeliverable mail, spearating mail to be forwarded and address changes.

(e) Time to strap mail into bundles or trays, and prepare relays.

All of the above times vary from day to day, and only time element (c) might be considered independent of the route size.

The office times for elements (a), (b), (d), (e) depend on the volume of mail, with letter casing time independent of flat casing time. The time for elements (d) and (e) depend on the total volume of letters and flats. Thus, the total office time, OT, is

$$OT = B + N \cdot [S_1 \bar{x} + S_f \bar{y} + S_r(\bar{x} + \bar{y}) + S_c(\bar{x} + \bar{y})]$$

where S_1 = standard casing time of letters in min./pcs. set by U.S.P.O. Department.

S_f = standard casing time of flats in min./pcs. set by U.S.P.O. Department.

S_r = standard time to bundle both letters and flats in min./pcs. set by U.S.P.O. Department.

S_c = standard time in mins./pcs. to work undeliverable mail. This time is estimated from the regression line superimposed on the scatter diagram in Figure 7.

B = constant average time in minutes for preparing route.

$\bar{x}$ = a normal random variable for the mean number of letters/stop for the residential area.

$\bar{y}$ = a normal random variable for the mean number of flats/

stop for the residential area.

N = the total number of possible stops (residences).

It is not possible to represent all the miscellaneous time duties explicitly in the above equation, such as personal needs, answering official communication and making address changes, therefore it is necessary to adjust the time element B to meet these requirements.

Street Time

The street time consist of the following:

(a) Time to travel to and from the route.

(b) Time to traverse the route without stopping.

(c) Time to deliver the stops with mail.

(d) Time for relay stops.

The travel time and relay time can be considered constant and independent of route size. Time element (b) is more critical for foot routes than mounted routes and will depend on the route size. Time element (c) will depend on the per cent of the N possible stops delivered. Thus, the total street time, ST, is

$$ST = D + N \cdot [S_n + S_d \cdot (1 - e^{-(\bar{x} + \bar{y})/k})]$$

where D = a constant average time in minutes to travel to and from the route, and make relay stops.

S_n = standard street time in mins./stop for walking or driving past a stop without delivering any mail.

S_d = standard street time in mins./stop for making a stop, (the time it requires to leave line of travel, deliver the mail, and return to line of travel).

The exponential term, in the above ST equation, was determined from the scatter diagram in Figure 5.

The total time, TT, for a route of size N is formed by adding street time and office time together. The equation for TT is simplified by the following substitutions:

$$K_1 = B + D$$
$$K_2 = N \cdot (S_1 + S_c + S_r)$$
$$K_3 = N \cdot (S_f + S_c + S_r)$$
$$K_4 = N \cdot (S_n + S_d)$$
$$K_5 = N \cdot S_d$$

Thus,

$$TT = K_1 + K_2\bar{x} + K_3\bar{y} + K_4 - K_5 e^{-(\bar{x} + \bar{y})/k}$$

where $\bar{x}$ an $\bar{y}$ are normal independent random variables.

It should be realized the above model, representing the total time necessary to perform the work functions for a route of N residences, is of a basic nature in that it does not claim to explicitly represent all intricate duties performed by the letter carrier. Basically, the model does provide a means of manipulating various parameters and variables and thus, enables one to observe the effect of such changes on the total time.

The Cost Model For Determining The Route Size

Once the model equation, TT, is developed, the question arises as to what size should the route be. This depend upon what criteria are used. Suppose, the criterion is to minimize the total expected cost of idle time and overtime.

Assuming that any work performed beyond eight hours is paid at an overtime rate, the following notation will represent the idle time and overtime respectively:

$$IT \leq A = 480 \text{ minutes} - K_1$$

$$OT \geq A = 480 \text{ minutes} - K_1$$

where K_1 is substracted since it is considered to be constant and independent of N and the random variables $\bar{x}$ and $\bar{y}$.

The total expected cost, TC, of idle time and overtime is

$$TC = C_I \int_0^A (A - T)g(T)dT + C_O \int_A^\infty (T - A)g(T)dT$$

where C_I = the cost of idle time, \$/min.

C_O = the cost of overtime, \$/min.

$g(T)$ = the density function of T, for a given N

T = $TT - K_1$

The objective is to determine the optimal "one-time" route size $\hat{N}$ which will minimize the total expected cost due to the probabilistic time T. Since T is a linear function of N, the total expected cost per stop, TCS, is obtained by dividing TC by N and making the transformation, $t = T/N$.

$$TCS = C_I \int_0^{A/N} (A/N - t)f(t)dt + C_O \int_{A/N}^\infty (t - A/N)f(t)dt$$

where $f(t)$ = the density function of the time per stop.

The TCS equation is minimized by differentiating TCS with respect to N, setting the results to zero and solving for N. Thus,

$$C_I(A/N^2)\int_0^{A/N} f(t)dt - C_O(A/N^2)\int_{A/N}^{\infty} f(t)dt = 0$$

Let $F(A/N) = \int_0^{A/N} f(t)dt$ and substitute into the above equation.

$$C_I \cdot F(A/N) - C_O \cdot (1 - F(A/N)) = 0$$

$$F(A/N) = C_O/(C_I + C_O)$$

The optimal N is such that the probability of idle time per stop equals

$$F(A/\hat{N}) = \int_0^{A/\hat{N}} f(t)dt = C_O/(C_I + C_O)$$

Therefore, the optimal N which minimizes TC is $\hat{N} = A/\hat{t}$.

Determination Of The Probability Distribution Function

The function T for a given N stops is

$$T = K_2\bar{x} + K_3\bar{y} + K_4 - K_5e^{-(\bar{x} + \bar{y})/k}$$

The distribution function of T, F(t), is required before the TC equation can be minimized. To determine F(t), the following procedure might be used. Let

$$u = K_2\bar{x} + K_3\bar{y}$$

$$v = (\bar{x} + \bar{y})/k$$

then

$$T = u - K_5e^{-v} + K_4$$

where u and v are normal random variables, but they are not independent.

The distribution function, $F(t)$, is equal to the probability that T is equal or less than t. That is

$$F(t) = P\{T \leq t\} = P\{u - K_5 \cdot e^{-v} + K_4 \leq t\} .$$

The distribution function is obtained by integrating over the region R such that $F(t) = \iint_R f(u,v)du\, dv$, where $f(u,v)$ is the joint density function of the random variables u and v, and R is the region composed of points (u,v) such that

$$u - K_5 e^{-v} + K_4 \leq t$$

That is

$$F(t) = \int_{-\infty}^{t} \left(\int_{-\infty}^{\phi(u,t)} f(u,v)dv \right) du$$

where

$$\phi(u,t) = \ln(1/K_5(u + K_4 - t)) .$$

Since $F(t)$ cannot be solved for t in the closed form, a table of $F(t)$ values would be required for different values of t.

The analytical approach to determine the distribution function is not as feasible, as far as the time and mathematics required, compared to a simulation approach.

The distribution function $F(t)$ can be evaluated approximately by the Monte Carlo technique. "In essence, the Monte Carlo technique consists of simulating an experiment to determine some probabilistic property of a population of objects or events by the use of random sampling applied to the components of the objects or events." (8).

To approximate the distribution function F(T), a computer program, written in Algol for the Burroughs 5500 computer was developed, and the experiment was run 1000 times. The program consisted of randomly selecting a positive $\bar{x}$ and $\bar{y}$ value from their respective normal density function, evaluating the function T for a given size N and then tabulating in five minute intervals, the value of T to estimate the distribution function.

Illustrative Example

The following example is for illustrative purposes only, since some of the time standards are estimated and have not been verified. Let B + D equal 95 minutes and then A is equal to 385 minutes.

The equation for T is determined with the following values:

N = 100 stops (residences)

S_c = 0.020 min./pc.

S_l = 1 min./18 pcs.

S_f = 1 min./8 pcs.

S_r = 1 min./70 pcs.

S_n = 0.27 min./stop

S_d = 0.55 min./stop

$$T = 9.00\,\bar{x} + 15.94\,\bar{y} + 82.00 - 55 \cdot e^{-(\bar{x} + \bar{y})/1.25}$$

The frequency histogram, Figure 8, of the total variable time per stop was obtained from the simulation program. The value of T was divided by 100 to express t in terms of minutes per stop. The estimated mean and standard deviation resulting from the above experiment are 0.9294 and 0.1309 minutes per stop respectively.

If C_o is equal to 1.5 C_I, then $C_o/(C_o + C_I)$ is equal to 0.60. The optimal N is determined such that the cumulative distribution, $F(A/\hat{N})$, equals 0.60. Observing the distribution function, the value of $\hat{t}$ was determined as 0.966 minutes per stop. Therefore, the optimal N for the route is

$$\hat{N} = A/\hat{t}$$

$$\hat{N} = 385/0.966$$

$$\hat{N} = 400 \text{ stops.}$$

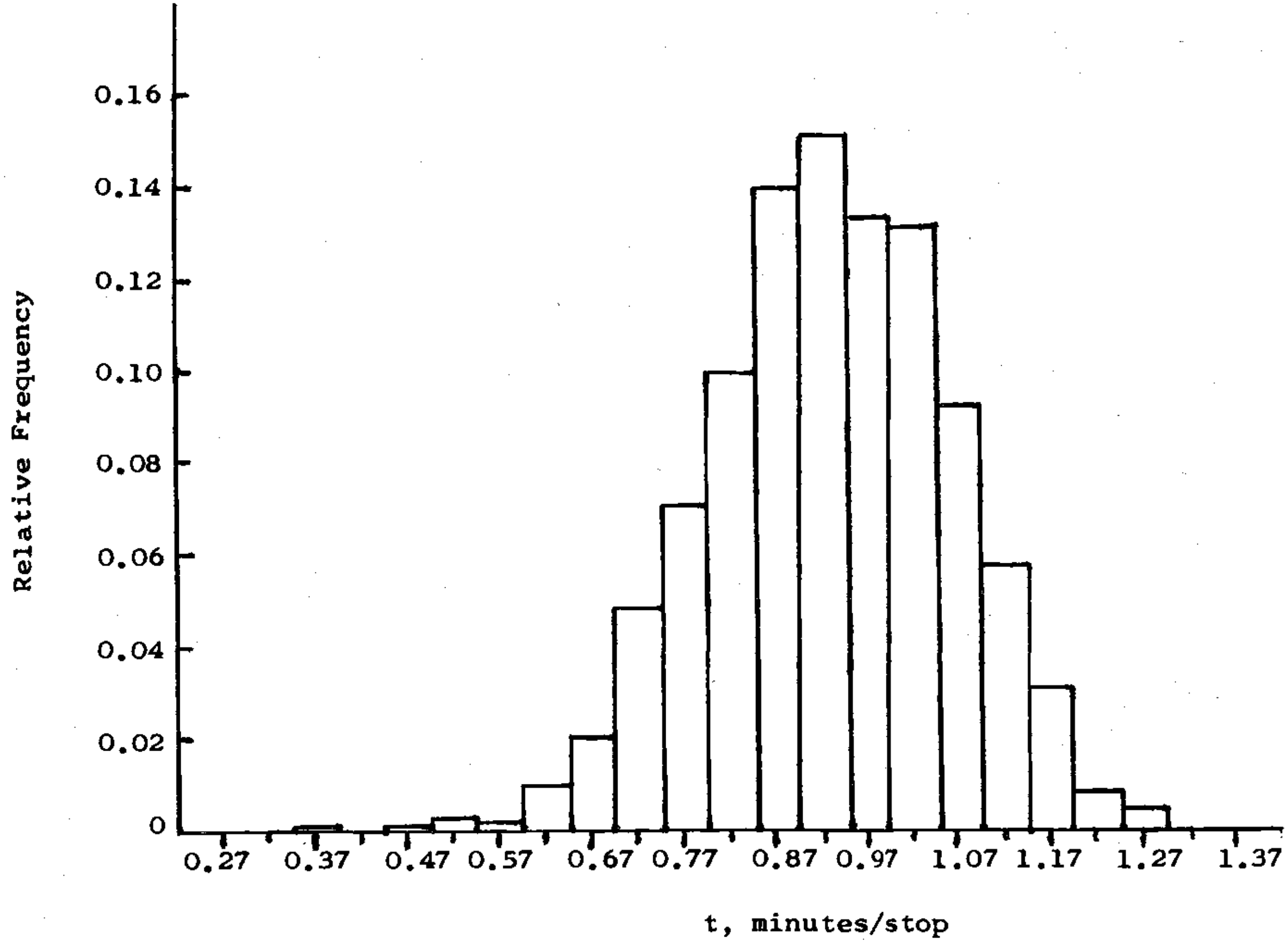

Figure 8. Frequency Histogram of Total Variable Time Per Stop from Simulation Program

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

While the preceding analysis is not as complete as could be desired, it is sufficient for the objective of the study. The conclusions drawn from the first part of this study are as follows:

1. The volume of letters and flats per day are independent.
2. The hypotheses that the distributions of the volume of letters per day and flats per day for the given residential area are normal distributions was not rejected at the 5 per cent significance level, using the χ^2 test for goodness of fit.
3. A relationship between the percentage of stops made and the total volume of mail can be estimated by an exponential model of the form $p_i = (1 - e^{-\bar{z}_i/1.25})$. Thus, a relationship between street time and office time can be estimated.
4. The standard time required to work undeliverable mail can be estimated by a linear model of the form $w_i = S_c \bar{z}_i$, where S_c is in minutes per piece allocated to work undeliverable mail; however, the residual variance is large so that there will be occurrences which will have large deviations from the estimated mean.
5. The hypotheses of equal means per week for the volume of letters and flats were not rejected, although a one week

sample could result in a 25 per cent difference of another one week sample.

The following conclusions are from the second part which dealt with the model. The total time required to work a residential route of N single family residences with similar delivery service for each residence is stated mathematically as follows:

$$TT = K_1 + K_2\bar{x} + K_3\bar{y} + K_4 - K_5 e^{-(\bar{x} + \bar{y})/k}$$

where

$$K_1 = B + D$$
$$K_2 = N(S_l + S_r + S_c)$$
$$K_3 = N(S_f + S_r + S_c)$$
$$K_4 = N(S_n + S_d)$$
$$K_5 = NS_d$$

and the above symbols are defined in Chapter V.

The standard times S_l, S_f, and S_r are determined by the U.S.P.O. Department and are considered to be the same for each route. Whereas, the other times included in the TT equation are characteristic of the residential area and are determined in such a manner as to include these traits. The variables $\bar{x}$ and $\bar{y}$ are assumed to be normal independent random variables representing the mean number of letters per stop and flats per stop respectively for the residential area.

Assuming that the work is performed at a constant rate and the criterion is to minimize the total expected cost of idle time and overtime, the optimal route size, $\hat{N}$, was determined by

$$F(A/\hat{N}) = \int_{0}^{A/\hat{N}} f(t)dt$$

$$F(A/\hat{N}) = C_O/(C_I + C_O)$$

where f(t) is the density function of the time per stop, and C_I and C_O are the cost of idle time and overtime respectively. Therefore, the optimal N is equal to $A/\hat{t}$, where $\hat{t}$ is determined from the above distribution function and A is the break even point between idle time and overtime.

The cost involved in estimating the standard times used in the model is not an expensive data collecting process. However, the actual cost effect of using the above procedure for determining the route size, instead of the present method, would require further investigation which is beyond the scope of this study.

Recommendations

In accordance with scope, limitations, and conclusions of this investigation, the following recommendations are offered:

1. An investigation of the present one week cluster sampling procedure for estimating the mail volume for a route compared to other sampling techniques could be a valuable contribution to the United States Post Office Department.
2. A study of the feasibility of establishing an incentive system for the letter carrier's job could prove to be very economical.
3. If standard times are not used in determining the route size, an interesting relationship would be the performance of a

carrier as a function of the total volume of mail.

4. An exponential model was developed in this study for estimating the percentage of stops receiving mail as a function of the total mail volume. If the data could be collected, one might investigate the possibility of a multi-regression model for estimating street time as a function of the percentage of stops, the volume of letters and the volume of flats.

APPENDIX

Table 7. Undeliverable Mail, Mean Number of Pieces Per Stop, and Percentage of Stops Made By Days

DATE 1967	Undel. Mail for 336 stops Class type, pieces 1st	2nd	3rd	minutes w_i *	pcs./stop $\bar{z}_i$	per cent p_i
July 18	25	0	21	16	2.32	90
19	14	3	5	7	2.36	94
20	20	4	10	13	1.64	66
21	19	7	3	12	1.60	72
22	12	7	13	13	1.80	80
24	12	10	20	17	1.32	60
25	13	10	11	14	2.56	86
26	14	5	31	19	2.58	98
27	26	9	12	19	3.82	96
28	20	7	22	19	2.70	84
29	17	4	33	20	2.62	86
31	14	6	14	13	1.70	68
Aug. 1	9	3	6	8	1.76	86
2	17	4	10	12	2.60	92
3	39	6	18	23	3.24	100
4	24	4	26	20	2.70	90
5	10	1	17	11	1.66	68
7	17	0	4	7	1.22	62
8	22	1	55	28	2.36	98
9	1	1	65	23	3.46	98
10	29	10	30	7	1.64	76
11	19	0	17	13	1.96	78
12	10	1	45	20	2.36	84
14	36	3	22	22	2.76	100
15	8	1	24	12	2.60	96
16	6	6	5	8	1.98	88
17	22	4	20	18	1.56	74
18	9	0	8	6	1.90	80
19	7	2	4	4	1.46	66
21	9	1	24	12	1.70	62

*w_i was calculated using time standards on page 8, items 9, 11, and 12.

<u>Calculation of the Sums of Squares Used in Table 3</u>

$$SS_{total} = \sum_i \sum_j X_{ij}^2 - W_{..}^2/N$$

$$= 66.59 - (42.32)^2/30$$

$$= 66.59 - 59.70$$

$$= 6.89$$

$$SS_{weeks} = \sum_j W_{.j}^2/n - W_{..}^2/N$$

$$SS_{weeks} = ((6.72)^2 + (9.90)^2 + (9.24)^2 + (9.26)^2 + (7.20)^2)/6-59.70$$

$$SS_{weeks} = 1.32$$

$$SS_{days} = \sum_i W_{i.}^2/k - W_{..}^2/N$$

$$SS_{days} = 60.92 - 59.70$$

$$SS_{days} = 1.22$$

$$SS_{error} = SS_{total} - SS_{weeks} - SS_{days}$$

$$SS_{error} = 6.89 - 1.32 - 1.22$$

$$SS_{error} = 4.35$$

Simulation Program For Determining Distribution Function

```
        BEGIN

FILE IN SV1 (2,10);
FILE OUT SV2 6(2,15);
REAL      X,Y,Z,OT,ST,TT,SUMTT,SQTT,MTT,STDTT,L,XX;
INTEGER   I,N,J, C ;
ARRAY     Q[O:1];
INTEGER ARRAY B[0:30] ;
FORMAT    FT1 (I10,4(F10.2),F20,2) ;
FORMAT    FT2 (///,2(F20.3)//);
FORMAT    FT3 (F10.1,I20,I20);
REAL PROCEDURE RAN;
        BEGIN
REAL      TH,TL;
DOUBLE(XX,O,Q[O], O x,←,TH,TL);
DOUBLE(TH,TL,ENTIER(TH),O,-,←,XX,TL);
RAN←XX;
        END;
REAL PROCEDURE ZETA ;
BEGIN INTEGER K;

REAL SUM;
SUM←O;
FOR K ←1 STEP 1 UNTIL 12 DO
SUM ← SUM +RAN ;
ZETA ← SUM -6;
END OF ZETA ;

WRITE (SV2[NO] ) ;
FILL Q [*] WITH OCT4444444444443;
XX←Q[O]/8*13;
          N ← 1000 ;
FOR I ← 1 STEP 1 UNTIL N DO
        BEGIN
LABEL   LX,LY ;

LY:     Y ← 0.787 +0.453xZETA ;
        IF Y< O THEN GO TO LY;
LX:     X ← 1.411 +0.488xZETA ;
        IF X< O THEN GO TO LX;
        Z ← X+Y ;
        OT ← 9xX +15.94xY    ;
        ST ← 82 - 55x(2.178*(-Z/1.25)) ;
        TT ← OT+ST ;
        J ← (TT - 27 ) / 5 ;
B[J]←B[J] + 1 ;
SUMTT ← SUMTT + TT ;
SQTT ← SQTT + TT*2 ;
```

```
WRITE (SV2,FT1,I,X,Y,OT,ST,TT );
        END;
MTT ← SUMTT/N ;
STDTT ← SQRT(SQTT/N - MTT*2) ;
WRITE (SV2,FT2,MTT ,STDTT );
FOR J ← 0 STEP 1 UNTIL 30 DO
WRITE (SV2,FT3, L ← 29.5 + 5xJ , B [ J], C ← C +B [ J] );
        END .
```

Computer Output

MEAN = 92.937 STD. DEV. = 13.087

INTERVALS	FREQUENCY	CUMULATIVE FREQUENCY
29.5	0	0
34.5	0	0
39.5	1	1
44.5	0	1
49.5	1	2
54.5	3	5
59.5	2	7
64.5	9	16
69.5	20	36
74.5	48	84
79.5	70	154
84.5	99	253
89.5	139	392
94.5	151	543
99.5	133	676
104.5	131	807
109.5	92	899
114.5	57	956
119.5	31	987
124.5	8	995
129.5	5	1000
134.5	0	1000
139.5	0	1000

SAMPLE DATA FORM

CARRIER: DATE:

COMMENTS:

DELIVERY #	LETTERS del.	FLATS del.	TOTAL PCS. del.		DELIVERY #	LETTERS del.	FLATS del.	TOTAL PCS. del.	
CASCADE AVE.					1551				
724					1609				
722					N. OLY. WAY				
698					1629				
694					1635				
654					1679				
597					1689				
645					1705				
ROGERS AVE.					1737				
1579					1752				
1609					1770				
1613					S. OLY. WAY				
1635					1785				
1669					1779				
1709					1769				
1735					1692				
1740					OLY. CIR.				
1730					1605				
1690					1569				
1656					1565				
1576					1558				
1536					1572				
CASCADE AVE					1598				
667					1606				
721					OLY. WAY				
747					1586				
OLY. WAY					1564				
1507					1532				
1511					1510				
TOTALS									

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